

Flavoured soft leptogenesis and natural values of the B term

Chee Sheng Fong

*C.N. Yang Institute for Theoretical Physics
State University of New York at Stony Brook
Stony Brook, NY 11794-3840, USA.
E-mail: fong@insti.physics.sunysb.edu*

M. C. Gonzalez-Garcia

*C.N. Yang Institute for Theoretical Physics
State University of New York at Stony Brook
Stony Brook, NY 11794-3840, USA and
Institució Catalana de Recerca i Estudis Avançats (ICREA)
Departament d'Estructura i Constituents de la Matèria and ICC-UB
Universitat de Barcelona, Diagonal 647, E-08028 Barcelona, Spain.
E-mail: concha@insti.physics.sunysb.edu*

Enrico Nardi

*INFN, Laboratori Nazionali di Frascati
C.P. 13, 100044 Frascati, Italy.
E-mail: enrico.nardi@lnf.infn.it*

J. Racker

*Departament d'Estructura i Constituents de la Matèria and ICC-UB
Universitat de Barcelona, Diagonal 647, E-08028 Barcelona, Spain.
E-mail: racker@ecm.ub.es*

ABSTRACT: We revisit flavour effects in soft leptogenesis relaxing the assumption of universality for the soft supersymmetry breaking terms. We find that with respect to the case in which the heavy sneutrinos decay with equal rates and equal CP asymmetries for all lepton flavours, hierarchical flavour configurations can enhance the efficiency by more than two orders of magnitude. This translates in more than three orders of magnitude with respect to the one-flavour approximation. We verify that lepton flavour equilibration effects related to off-diagonal soft slepton masses are ineffective for damping these large enhancements. We show that soft leptogenesis can be successful for unusual values of the relevant parameters, allowing for $B \sim \mathcal{O}(\text{TeV})$ and for values of the washout parameter up to $m_{\text{eff}}/m_* \sim 5 \times 10^3$.

KEYWORDS: Leptogenesis, Supersymmetry, Beyond Standard Model, Cosmology of Theories beyond the SM.

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1. Introduction

The discovery of neutrino oscillations makes leptogenesis a very attractive solution to the baryon asymmetry problem [1,2]. In the *standard* type I seesaw framework [3], the singlet heavy neutrinos have lepton number violating Majorana masses and when decay out of equilibrium produce dynamically a lepton asymmetry which is partially converted into a baryon asymmetry due to fast sphaleron processes.

For a hierarchical spectrum of the $SU(2)$ singlets Majorana neutrinos, successful leptogenesis requires generically quite heavy singlet neutrino masses [4], of order $M > 2.4(0.4) \times 10^9$ GeV for vanishing (thermal) initial neutrino densities [4,5] (although flavour effects [6–9] and/or extended scenarios [10,11] may affect this limit). Low-energy supersymmetry can be invoked to naturally stabilize the hierarchy between this new scale and the electroweak one. This, however, introduces a certain conflict between the gravitino bound on the reheat temperature and the thermal production of the heavy singlets neutrinos [12]. A way out of this conflict is provided by resonant leptogenesis [13]. In this scenario, the heavy Majorana neutrinos are nearly degenerate in mass which makes the self energy contributions to the CP asymmetries resonantly enhanced, thus allowing for successful leptogenesis at much lower temperatures.

Once supersymmetry has been introduced, leptogenesis is induced also in singlet sneutrino decays. If supersymmetry is not broken, the order of magnitude of the asymmetry and the basic mechanism are the same as in the non-supersymmetric case. However, as shown in Refs. [14,15], supersymmetry-breaking terms can induce effects which are essentially different from the neutrino ones. In brief, soft supersymmetry-breaking terms involving the singlet sneutrinos remove the mass degeneracy between the two real sneutrino states of a single neutrino generation, and provide new sources of lepton number and CP violation.

In this case, as for the case of resonant leptogenesis, it is the sneutrino self-energy contributions to the CP asymmetries which are resonantly enhanced. As a consequence, the mixing between the sneutrino states can generate a sizable CP asymmetry in their decays. This scenario was termed “soft leptogenesis” [15].

Altogether it was found that the asymmetry is large for a Majorana neutrino mass scale relatively low, in the range $10^5 - 10^8$ GeV. A sizable part of this range lies below the reheat temperature limits, what solves the cosmological gravitino problem. However, in order to generate enough asymmetry the lepton-violating soft bilinear coupling, B , responsible for the sneutrino mass splitting, has to be unconventionally small [14–17] *.

In Refs. [18, 21] the possibility of soft leptogenesis generated by CP violation in the decay of the heavy sneutrinos, and in the interference of mixing and decay was considered. These new sources of CP violation (the so called “new ways to soft leptogenesis” [21]) are induced by vertex corrections due to gaugino soft supersymmetry-breaking masses. In all these processes, at first order in soft breaking terms and at $T = 0$, the CP asymmetries for the decays into fermions and bosons are equal in magnitude and opposite in sign [14, 15, 18]. Therefore, assuming equality in the time evolutions of the fermion and scalar lepton asymmetries (that at sufficiently low temperatures is certainly correct because of super-equilibration of the particle-antiparticle chemical potentials [22]) in the $T = 0$ limit an exact cancellation occurs between the lepton asymmetry produced in the fermionic and bosonic channels [14, 15, 18]. Thermal effects, thus, play a fundamental role in soft leptogenesis: final-state Fermi blocking and Bose stimulation as well as effective masses for the particle excitations in the plasma break supersymmetry and effectively spoil the cancellation.

In Refs. [14, 15, 21] soft leptogenesis was addressed within the ‘one-flavour’ approximation. This one-flavour approximation is rigorously correct only when the interactions mediated by charged lepton Yukawa couplings are out of equilibrium. This is not the case in soft leptogenesis since, as mentioned above, successful leptogenesis in this scenario requires a relatively low mass scale for the singlet neutrinos. Thus the characteristic temperature is such that the rates of processes mediated by the τ and μ Yukawa couplings are not negligible, implying that the effects of lepton flavours must be taken into account. The impact of flavour in thermal leptogenesis in the context of standard see-saw leptogenesis has been investigated in great detail in several papers [6, 7, 9, 11, 13, 23–26]. In general, the result of including flavour effects is that the produced baryon asymmetry can get considerably enhanced, and some of the constraints on the required value of the heavy Majorana neutrino and sneutrino masses can be relaxed. The effects of spectator processes, that are fast processes that do not violate lepton number but that can still have an impact on lepton asymmetry production, was analyzed in [27, 28]. It was found that within the standard leptogenesis scenario the size of the related corrections is at most of $\mathcal{O}(1)$, and less important than effects related to the lepton flavours. A quite general characteristic of models for new physics, like for example the MSSM, is the presence of new sources of lepton flavour violation, that are not suppressed by the light neutrino masses. As has been highlighted in ref. [29], at sufficiently low temperatures the related new effects can give rise

*Flavour effects [18] and extended scenarios [19, 20] may alleviate the unconventionally-small- B problem.

to lepton flavour equilibration (LFE), and when this occurs all dynamical flavour effects get effectively damped.

Ref. [16] introduced flavour and spectator effects in the soft leptogenesis scenario under the restrictive assumption of universal trilinear couplings. The authors found that within that context, these effects could enhance the efficiency by $\mathcal{O}(30)$.

In this work we revisit the impact of flavour in soft leptogenesis. In particular we extend the analysis of Refs. [16,29] by relaxing the assumption of universal trilinear couplings. We find that under these conditions flavour effects can play an even more important role, enhancing the leptogenesis efficiency by more than three orders of magnitude with respect to the one-flavour approximation. Given the importance that flavour effects can acquire in soft leptogenesis with non-universal soft supersymmetry breaking terms, we also quantify the LFE effects associated with off-diagonal soft breaking masses for the scalar lepton doublets. We find that in most part of the supersymmetry (SUSY) parameter space that is relevant for soft leptogenesis, the large flavour enhancements can survive LFE effects.

The outline of the paper is as follows. Section 2 summarizes the soft leptogenesis scenario for the general flavour structure of the relevant trilinear couplings, and presents the corresponding CP violation asymmetries. In Sec. 3 we review the flavour changing processes associated with off-diagonal soft breaking masses for the scalar lepton doublets, and we discuss the temperature regime in which LFE becomes relevant. In Sec. 4 we present the relevant Boltzmann Equations (BE) that describe the production of the lepton asymmetry in this scenario. To quantify the achievable flavour enhancements as well as the possible impact of LFE effects, we then solve the BE for different flavour configurations and in different temperature regimes. In Sec. 5 we discuss the possible connection with observable lepton flavour violation phenomena at low energies. Finally, in Sec. 6 we summarize our results and draw the conclusions.

2. Soft Leptogenesis Lagrangian and CP Asymmetries

The supersymmetric see-saw model can be described by the superpotential:

$$W = \frac{1}{2}M_{ij}N_iN_j + Y_{ik}\epsilon_{\alpha\beta}N_iL_k^\alpha H^\beta, \quad (2.1)$$

where i, j are the generation indices of heavy Majorana ‘right-handed’ (RH) neutrinos, $k = 1, 2, 3$ are the lepton flavour indices, and N_i, L_k, H are the chiral superfields for the RH neutrinos, the left-handed (LH) lepton doublets and the Higgs doublets, with $\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}$ and $\epsilon_{12} = +1$.

The relevant soft supersymmetry breaking terms involving the RH sneutrinos \widetilde{N}_i and $SU(2)$ gauginos $\widetilde{\lambda}_2^a$ are given by [†]

$$\mathcal{L}_{soft} = - \left(AZ_{ik}\epsilon_{\alpha\beta}\widetilde{N}_i\widetilde{\ell}_k^\alpha h_2^\beta + \frac{1}{2}B_{ij}M_{ij}\widetilde{N}_i\widetilde{N}_j + \frac{1}{2}m_2\widetilde{\lambda}_2^a P_L\widetilde{\lambda}_2^a + \text{h.c.} \right). \quad (2.2)$$

[†]The effect of $U(1)$ gauginos can be included in similar form.

The sneutrino and anti-sneutrino states mix, giving rise to the mass eigenstates:

$$\begin{aligned}\tilde{N}_{+i} &= \frac{1}{\sqrt{2}}(e^{i\Phi/2}\tilde{N}_i + e^{-i\Phi/2}\tilde{N}_i^*), \\ \tilde{N}_{-i} &= \frac{-i}{\sqrt{2}}(e^{i\Phi/2}\tilde{N}_i - e^{-i\Phi/2}\tilde{N}_i^*),\end{aligned}\tag{2.3}$$

where $\Phi \equiv \arg(BM)$, that correspond to the mass eigenvalues

$$M_{ii\pm}^2 = M_{ii}^2 \pm |B_{ii}M_{ii}|.\tag{2.4}$$

The Lagrangian for the interactions involving the RH sneutrinos $\tilde{N}_{\pm i}$, the RH neutrinos N_i and the $SU(2)$ gauginos $\tilde{\lambda}_2$, with the (s)leptons and the Higgs(inos) can be written as:

$$\begin{aligned}\mathcal{L}_{int} &= -\epsilon_{\alpha\beta} \left\{ \frac{1}{\sqrt{2}}\tilde{N}_{+i} \left[Y_{ik}\tilde{h}^\beta P_L \ell_k^\alpha + (AZ_{ik} + M_i Y_{ik})\tilde{\ell}_k^\alpha h_2^\beta \right] \right. \\ &\quad \left. + \frac{i}{\sqrt{2}}\tilde{N}_{-i} \left[Y_{ik}\tilde{h}^\beta P_L \ell_k^\alpha + (AZ_{ik} - M_i Y_{ik})\tilde{\ell}_k^\alpha h_2^\beta \right] + Y_{ik}\tilde{h}^\beta P_L N_i \tilde{\ell}_k^\alpha + Y_{ik}\bar{N}_i P_L \ell_k^\alpha h_2^\beta \right\} \\ &\quad - g_2 \left(\tilde{\lambda}_2^\pm P_L (\sigma_1)_{\alpha\beta} \ell_k^\alpha \tilde{\ell}_k^{\beta*} - \frac{1}{\sqrt{2}}\tilde{\lambda}_2^0 P_L (\sigma_3)_{\alpha\beta} \ell_k^\alpha \tilde{\ell}_k^{\beta*} \right. \\ &\quad \left. + \tilde{h}^\alpha P_L (\sigma_1)_{\alpha\beta} \tilde{\lambda}_2^\pm h_2^{\beta*} - \frac{1}{\sqrt{2}}\tilde{h}^\alpha P_L (\sigma_3)_{\alpha\beta} \tilde{\lambda}_2^0 h_2^{\beta*} \right) + \text{h.c.},\end{aligned}\tag{2.5}$$

where $\ell_k^T = (\nu_k, \ell_k^-)$, $\tilde{\ell}_k^T = (\tilde{\nu}_k, \tilde{\ell}_k^-)$ are the lepton and slepton doublets, and $h_2^T = (h_2^+, h_2^0)$, $\tilde{h}^T = (\tilde{h}^-, \tilde{h}^0)$ are the Higgs and Higgsino doublets. $\tilde{\lambda}_2^\pm$ denotes $\tilde{\lambda}_2^+$ for $(\alpha\beta) = (01)$ and $\tilde{\lambda}_2^-$ for $(\alpha\beta) = (10)$ with $\sigma_{1,3}$ being the Pauli matrices, and $P_{L,R}$ are respectively the left and right projection operator.

All the parameters appearing in the superpotential Eq. (2.1) and in the Lagrangian Eq. (2.2) (and equivalently in the first two lines of Eq. (2.5)) are in principle complex quantities. However, superfield phase redefinition allows to remove several complex phases. Here for simplicity, we will concentrate on soft leptogenesis arising from a single sneutrino generation $i = 1$ and in what follows we will drop that index ($Y_k \equiv Y_{1k}$, $Z_k \equiv Z_{1k}$, $B = B_{11}$, etc.). Thus we will be only interested in the physical phases involving the sneutrinos of the first generation. After superfield phase rotations, the relevant Lagrangian terms restricted to $i = 1$ are characterized by only four independent physical phases, that are

$$\phi_{Ak} = \arg(Z_k Y_k^* A B^*), \quad (k = 1, 2, 3)\tag{2.6}$$

$$\phi_g = \frac{1}{2}\arg(B m_2^*),\tag{2.7}$$

which we choose to assign respectively to the slepton-Higgs-sneutrino trilinear soft breaking terms, and to the gaugino coupling operators respectively. In what follows we will keep track of these physical phases explicitly and, differently from the convention used in Eqs. (2.1), (2.2) and (2.5), we will leave understood (unless when explicitly stated in the text) that all the other parameters Y_k , Z_k , B , m_2 , etc. correspond to real and positive values.

Neglecting supersymmetry breaking effects in the RH sneutrino masses and in the vertex, the total singlet sneutrino decay width is given by

$$\Gamma_{\tilde{N}_+} = \Gamma_{\tilde{N}_-} \equiv \Gamma_{\tilde{N}} = \frac{M}{4\pi} \sum_k Y_k^2 \equiv \frac{m_{\text{eff}} M^2}{4\pi v_u^2}, \quad (2.8)$$

where v_u is the vacuum expectation value of the up-type Higgs doublet, $v_u = v \sin \beta$ (with $v=174$ GeV), and $m_{\text{eff}} \equiv \sum_k Y_k^2 v_u^2 / M$ is the \tilde{N}_\pm decay parameter. It is related with the washout parameter K as $K = \Gamma_{\tilde{N}_+} / H(M) = m_{\text{eff}} / m_*$ where the equilibrium neutrino mass is $m_* = \sqrt{\frac{\pi g^*}{45}} \times \frac{8\pi^2 v_u^2}{m_P} \sim 10^{-3}$ eV with g^* the total number of relativistic degrees of freedom ($g^* = 228.75$ in the MSSM) and m_P the Planck mass.

Equation (2.2) leads to three contributions to the CP asymmetry in $\tilde{N}_\pm \rightarrow l_k \tilde{h}, \tilde{l}_k h$ decays [18,21]: ϵ_k^S arising from self-energy diagrams induced by the bilinear B term, ϵ_k^V arising from vertex diagrams induced by the gaugino masses, and ϵ_k^I which is due to interference of self-energy and vertex. They can be written as

$$\epsilon_k^S(T) = -P_k \frac{Z_k}{Y_k} \sin \phi_{Ak} \frac{A}{M} \frac{4B\Gamma}{4B^2 + \Gamma^2} \Delta_{BF}(T), \quad (2.9)$$

$$\begin{aligned} \epsilon_k^V(T) = & -P_k \frac{3\alpha_2}{4} \frac{m_2}{M} \ln \frac{m_2^2}{m_2^2 + M^2} \Delta_{BF}(T) \\ & \times \left\{ \frac{Z_k}{Y_k} \left[\sin \phi_{Ak} \frac{A}{M} \cos(2\phi_g) + \cos \phi_{Ak} \frac{A}{M} \sin(2\phi_g) \right] - \frac{B}{M} \sin(2\phi_g) \right\}, \end{aligned} \quad (2.10)$$

$$\epsilon_k^I(T) = P_k \frac{Z_k}{Y_k} \frac{3\alpha_2}{2} \sin \phi_{Ak} \frac{A}{M} \left(\ln \frac{m_2^2}{m_2^2 + M^2} \right) \cos(2\phi_g) \frac{\Gamma^2}{4B^2 + \Gamma^2} \Delta_{BF}(T), \quad (2.11)$$

where

$$\Delta_{BF}(T) = \frac{c^s(T) - c^f(T)}{c^s(T) + c^f(T)} \quad (2.12)$$

is the thermal factor associated to the difference between the phase-space factors for the scalar and fermionic channels, that vanishes in the zero temperature limit $\Delta_{BF}(T=0) = 0$. As long as we neglect the zero temperature slepton masses and small Yukawa couplings, $c^f(T)$ and $c^s(T)$ are flavour independent and they are the same for \tilde{N}_\pm . In the approximation in which \tilde{N}_\pm decay at rest, the $c^{f,s}(T)$ functions are given by:

$$c^f(T) = (1 - x_\ell - x_{\tilde{h}}) \lambda(1, x_\ell, x_{\tilde{h}}) [1 - f_\ell^{eq}] [1 - f_{\tilde{h}}^{eq}], \quad (2.13)$$

$$c^s(T) = \lambda(1, x_h, x_{\tilde{\ell}}) [1 + f_h^{eq}] [1 + f_{\tilde{\ell}}^{eq}], \quad (2.14)$$

where

$$f_{h,\tilde{\ell}}^{eq} = \frac{1}{\exp[E_{h,\tilde{\ell}}/T] - 1}, \quad (2.15)$$

$$f_{\tilde{h},\ell}^{eq} = \frac{1}{\exp[E_{\tilde{h},\ell}/T] + 1}, \quad (2.16)$$

are respectively the Bose-Einstein and Fermi-Dirac equilibrium distributions, and

$$E_{\ell, \tilde{h}} = \frac{M}{2}(1 + x_{\ell, \tilde{h}} - x_{\tilde{h}, \ell}), \quad E_{h, \tilde{\ell}} = \frac{M}{2}(1 + x_{h, \tilde{\ell}} - x_{\tilde{\ell}, h}), \quad (2.17)$$

$$\lambda(1, x, y) = \sqrt{(1 + x - y)^2 - 4x}, \quad x_a \equiv \frac{m_a(T)^2}{M^2}. \quad (2.18)$$

The thermal masses for the relevant supersymmetric degrees of freedom are [30]:

$$m_h^2(T) = 2m_{\tilde{h}}^2(T) = \left(\frac{3}{8}g_2^2 + \frac{1}{8}g_Y^2 + \frac{3}{4}\lambda_t^2 \right) T^2, \quad (2.19)$$

$$m_{\tilde{\ell}}^2(T) = 2m_{\ell}^2(T) = \left(\frac{3}{8}g_2^2 + \frac{1}{8}g_Y^2 \right) T^2, \quad (2.20)$$

where g_2 and g_Y are the $SU(2)$ and $U(1)$ gauge couplings, and λ_t is the top Yukawa coupling, renormalized at the appropriate energy scale.

In Eq.(2.9) we have defined the Yukawa flavour projectors:

$$P_k \equiv \frac{Y_k^2}{\sum_j Y_j^2} \quad (2.21)$$

which are constrained by the condition

$$\sum_k P_k = 1 \quad \longrightarrow \quad 0 \leq P_k \leq 1. \quad (2.22)$$

Regarding the flavour structure of the soft terms relevant for flavoured soft leptogenesis, we can distinguish two general possibilities:

1. *Universal soft supersymmetry breaking terms.* This case is realized in supergravity and gauge mediated SUSY-breaking models (when the renormalization group running of the parameters is neglected), and in our notation corresponds to set

$$Z_k = Y_k. \quad (2.23)$$

This *Universal Trilinear Scenario* (UTS) is the one that was considered so far in the literature on flavoured soft leptogenesis [16, 18]. In this case the only flavour structure arises from the Yukawa couplings and both the total CP asymmetries $\epsilon^k = \epsilon_k^S + \epsilon_k^V + \epsilon_k^I$ and the corresponding washout terms, that are generically denoted as W_k , are proportional to the same flavour projections, yielding:

$$\frac{\epsilon^e}{W_e} = \frac{\epsilon^\mu}{W_\mu} = \frac{\epsilon^\tau}{W_\tau}. \quad (2.24)$$

Furthermore, as seen in Eq.(2.6) there is a unique phase for the trilinear couplings $\phi_{Ak} \equiv \phi_A = \arg(AB^*)$.

2. *General soft supersymmetry breaking terms.* In this case the most general form for the soft-SUSY breaking terms is allowed, only subject to the phenomenological constraints from limits on flavour changing neutral currents (FCNC) and from lepton flavour violating (LFV) processes. The trilinear soft-breaking terms are not aligned with the corresponding

Yukawa couplings, and Eq. (2.24) does not hold. Studying this scenario can be rather involved due to the large dimensionality of the relevant parameter space. Therefore we will introduce a drastic simplification that, while it can still capture some of the main features of the general case, it allows to carry out an analysis in terms of the same number of independent parameters than in case 1.

Let us note that the CP asymmetries become flavour independent (except for the last term in Eq.(2.10)) if

$$Z_k = \frac{\sum_j |Y_j|^2}{3Y_k^*}, \quad (2.25)$$

where we have kept Z and Y explicitly as complex numbers. Eq. (2.25) yields $\epsilon^k = \epsilon/3$ for each flavour, and from Eq. (2.6) we see that, since $Z_k Y_k^*$ is real, also in this case there is a unique phase for the trilinear couplings $\phi_{Ak} \equiv \phi_A = \arg(AB^*)$. The normalization factor of $1/3$ in Eq. (2.25) has been introduced so that both Eq.(2.23) and Eq. (2.25) yield the same total asymmetry $\sum_k \epsilon_k = \epsilon$. In what follows we will refer to this case as the *Simplified Misaligned Scenario* (SMS). Our SMS of course does not correspond to a completely general scenario, and for example, due to the reduction in the number of independent physical phases implied by Eq. (2.25), it excludes the possibility of having flavour asymmetries of opposite signs, with $|\epsilon^k| > |\epsilon|$ for some, or even for all, flavours. The reader should thus keep in mind that enhancements of the final lepton asymmetry even larger than the ones we will find within the SMS are certainly possible.

3. Lepton Flavour Equilibration

In the basis where charged lepton Yukawa couplings are diagonal, the SUSY breaking slepton masses read

$$\mathcal{L}_{soft} \supset \tilde{m}_{ij}^2 \tilde{\ell}_i^* \tilde{\ell}_j. \quad (3.1)$$

The off-diagonal slepton masses $\tilde{m}_{i \neq j}^2$ affect the flavour composition of the slepton mass eigenstates so generically we can write

$$\tilde{\ell}_i^{(int)} = R_{ij} \tilde{\ell}_j \quad (3.2)$$

where R_{ij} is a unitary rotation matrix. In this basis the corresponding slepton-gaugino interactions in Eq.(2.5) are

$$\begin{aligned} \mathcal{L}_{\tilde{\lambda}, \tilde{\ell}} = & -g_2 (\sigma_1)_{\alpha\beta} \overline{\tilde{\lambda}}_2^\pm P_L \ell_i^\alpha R_{ij}^* \tilde{\ell}_j^{\beta*} - \frac{g_2}{\sqrt{2}} (\sigma_3)_{\alpha\beta} \overline{\tilde{\lambda}}_2^0 P_L \ell_i^\alpha R_{ij}^* \tilde{\ell}_j^{\beta*} \\ & - \frac{g_Y}{\sqrt{2}} \delta_{\alpha\beta} \overline{\tilde{\lambda}}_1 Y_\ell P_L \ell_i^\alpha R_{ij}^* \tilde{\ell}_j^{\beta*} + \text{h.c.} , \end{aligned} \quad (3.3)$$

where $Y_\ell = -1$ is the hypercharge of the left-handed lepton doublets. The mixing matrix can be expressed in terms of the off-diagonal slepton masses as:

$$\begin{aligned} R_{ij} &\sim \delta_{ij} + \frac{\tilde{m}_{ij}^2}{h_i^2 T^2} \\ &= \delta_{ij} + \frac{\tilde{m}_{ij}^2 v^2 \cos^2 \beta}{m_i^2 M^2} z^2, \end{aligned} \quad (3.4)$$

where in the first line $h_i > h_j$ is the relevant charged Yukawa coupling that determines at leading order the thermal mass splittings of the sleptons, v in the second line is the electroweak symmetry breaking VEV with $v^2 = v_u^2 + v_d^2 \simeq 174 \text{ GeV}$, $z \equiv \frac{M}{T}$ where T is the temperature and M the mass of the RH neutrino, and $m_i \equiv m_{\ell_i}(T=0)$ is the zero temperature mass for the lepton ℓ_i . In what follows, for simplicity we construct the R_{ij} entries in such a way that they are flavour independent quantities. We assume $\tilde{m}_{i\tau} = \tilde{m}_{od}$ (for $i = e, \mu$) and $\tilde{m}_{e\mu} = \tilde{m}_{od} \frac{m_\mu}{m_\tau}$, where \tilde{m}_{od} is a unique off-diagonal soft-mass parameter. We thus obtain for $(ij) = (e\tau), (\mu\tau), (e\mu)$:

$$R_{ij} \sim \delta_{ij} + \frac{\tilde{m}_{od}^2 v^2 \cos^2 \beta}{m_\tau^2 M^2} z^2, \quad (3.5)$$

where m_τ is the mass of the tau lepton.

$\mathcal{L}_{\tilde{\lambda}, \tilde{l}}$ in Eq. (3.3) induces lepton flavour violating slepton scatterings through the exchange of $SU(2)$ and $U(1)_Y$ gauginos. There are two possible t-channel scatterings $\ell_i P \leftrightarrow \tilde{\ell}_j \tilde{P}$, $\ell_i \tilde{P} \leftrightarrow \tilde{\ell}_j P$ and one s-channel scattering $\ell_i \tilde{\ell}_j^* \leftrightarrow P \tilde{P}^*$ (we denote P as fermions and \tilde{P} as scalars). For processes mediated by $SU(2)$ gauginos $P = \ell, q, \tilde{h}$, while when mediated by $U(1)_Y$ gaugino one must include the $SU(2)$ singlet states $P = e, u, d$ as well. The corresponding reduced cross sections read:

$$\begin{aligned} \hat{\sigma}_{t1,G}^{ij}(s) &= \sum_P \frac{g_G^4 |R_{ij}|^2 \Pi_P^G}{8\pi} \left[\left(\frac{2m_{\lambda_G}^2}{s} + 1 \right) \ln \left| \frac{m_{\lambda_G}^2 + s}{m_{\lambda_G}^2} \right| - 2 \right], \\ \hat{\sigma}_{t2,G}^{ij}(s) &= \sum_P \frac{g_G^4 |R_{ij}|^2 \Pi_P^G}{8\pi} \left[\ln \left| \frac{m_{\lambda_G}^2 + s}{m_{\lambda_G}^2} \right| - \frac{s}{m_{\lambda_G}^2 + s} \right], \\ \hat{\sigma}_{s,G}^{ij}(s) &= \sum_P \frac{g_G^4 |R_{ij}|^2 \Pi_P^G}{16\pi} \left(\frac{s}{s - m_{\lambda_G}^2} \right)^2, \end{aligned} \quad (3.6)$$

where Π_P^G counts the numbers of degrees of freedom of the particle P (isospin, quark flavours and color) involved in the scatterings mediated by the $SU(2)$ ($G = 2$) and $U(1)_Y$ ($G = Y$) gauginos respectively. In this last case the hypercharges Y_ℓ and Y_P are also included in Π_P^Y . If the flavour changing scatterings in Eq. (3.6) are fast enough, they will lead to lepton flavour equilibration, and damp all leptogenesis flavour effects [29][‡].

[‡]See Ref. [31] for some particular effects associated with lepton flavour violating processes in scenarios with vanishing total CP asymmetry.

The values of \tilde{m}_{od} for which this occurs can be estimated by comparing the LFE scattering rates and the $\Delta L = 1$ washout rates. Since the dominant $\Delta L = 1$ contribution arises from inverse decays, the terms to be compared are:

$$\begin{aligned}\bar{\Gamma}_{\text{LFE}}(T) &\equiv \frac{\gamma_{\text{LFE}}(T)}{n_L^c(T)} \equiv \frac{1}{n_L^c(T)} \sum_{G,P} \Pi_P^G (\gamma_{t1,G}^{ij} + \gamma_{t2,G}^{ij} + \gamma_{s,G}^{ij}) \\ &= \frac{1}{n_L^c(T)} \frac{T}{64\pi} \sum_G \int ds \left[\hat{\sigma}_{t1,G}^{ij}(s) + \hat{\sigma}_{t2,G}^{ij}(s) + \hat{\sigma}_{s,G}^{ij}(s) \right] \sqrt{s} \mathcal{K}_1 \left(\frac{\sqrt{s}}{T} \right), \quad (3.7)\end{aligned}$$

$$\bar{\Gamma}_{\text{ID}}(T) \equiv \frac{\gamma_{\tilde{N}}(T)}{n_L^c(T)} = \frac{n_{\tilde{N}}^{eq}(T)}{n_L^c(T)} \frac{\mathcal{K}_1(z)}{\mathcal{K}_2(z)} \Gamma_{\tilde{N}}, \quad (3.8)$$

where the γ^{ij} in the first line represent the thermally averaged LFE reactions for one degree of freedom of the P -particle, $\mathcal{K}_{1,2}(z)$ are the modified Bessel function of the second kind of order 1 and 2, $\Gamma_{\tilde{N}}$ is the zero temperature width Eq. (2.8), and $n_{\tilde{N}}^{eq}$ is the equilibrium number density for \tilde{N} while $n_L^c = T^3/2$ is the relevant density factor appearing in the washouts (see next section for more details). In evaluating the reaction densities above we have not included the thermal masses, and we have neglected Pauli-blocking and stimulated emission as well as the relative motion of the particles with respect to the plasma.

LFE scattering reaction densities have a different T dependence with respect to the Universe expansion and to the decay rates. While for the expansion $H(T) \sim T^2$, for LFE processes we have $\bar{\Gamma}_{\text{LFE}} \sim T^{-3}$. This means that the ratio $\bar{\Gamma}_{\text{LFE}}/H \sim 1/T^5$, and thus once LFE reactions have attained thermal equilibrium, they will remain in thermal equilibrium also at lower temperatures. In contrast, $\bar{\Gamma}_{\text{ID}}$ first increases till reaching a maximum, but then decreases exponentially $\sim e^{-M/T}$ dropping out of equilibrium at temperatures not much below $T \sim M$. The relevant temperature where we should compare the rates of these interactions is when the inverse decay rate $\bar{\Gamma}_{\text{ID}}$ becomes slower than the expansion rate of the Universe H , that is when the lepton asymmetry starts being generated from the out-of-equilibrium \tilde{N}_{\pm} decays. We define z_{dec} as $\bar{\Gamma}_{\text{ID}}(z_{dec}) = H(z_{dec})$. LFE is expected to be quite relevant for flavoured leptogenesis when the following condition is verified:

$$\bar{\Gamma}_{\text{LFE}}(z_{dec}) \geq \bar{\Gamma}_{\text{ID}}(z_{dec}) = H(z_{dec}). \quad (3.9)$$

In this case, LFE processes are in equilibrium since the very onset of the era of out-of-equilibrium decays, and due to its temperature dependence it is guaranteed that they will remain in equilibrium until leptogenesis is over.

In the left panel of Fig. I we plot the ratio $\bar{\Gamma}_{\text{ID}}(z_{dec})/H(z_{dec})$ as a function of \tilde{m}_{od} for $m_{\text{eff}} = 0.1$ eV (defined in Eq. (2.8)), $\tan \beta = 30$, and for different values of M . From the figure we can read the characteristic value of \tilde{m}_{od} for which LFE becomes relevant. Notice that the dominant dependence on $\tan \beta \sim 1/\cos \beta$ ($\tan \beta \gg 1$) arises due to $v \cos \beta = v_d$ in Eq. (3.5). Thus the results from other values of $\tan \beta$ can be easily read from the figure by rescaling $\tilde{m}_{od}^{\beta} = \tilde{m}_{od}^{\text{fig}}/(30 \cos \beta)$.

Since we are interested in the dynamics of lepton flavours, to be more precise about LFE effects we should in fact consider the temperature z_{dec}^k at which the inverse decay rate

for one specific flavour goes out of equilibrium, that can be defined through $\bar{\Gamma}_{\text{ID}}^k(z_{\text{dec}}^k) = P_k \bar{\Gamma}_{\text{ID}}(z_{\text{dec}}^k) = H(z_{\text{dec}}^k)$. Let's assume $P_a < P_b < P_c$ which implies $z_{\text{dec}}^a < z_{\text{dec}}^b < z_{\text{dec}}^c$. In other words, assuming that the lepton doublet ℓ_a is the most weakly coupled to \tilde{N}_\pm , $\bar{\Gamma}_{\text{ID}}^a$ will go out of equilibrium first, and then $\bar{\Gamma}_{\text{ID}}^b$ and $\bar{\Gamma}_{\text{ID}}^c$ will follow. Hence, for given values of m_{eff} and M , the minimum value $\tilde{m}_{\text{od}}^{\text{min}}$ for which LFE effects start being important is given by the following condition:

$$\bar{\Gamma}_{\text{LFE}}(z_{\text{dec}}^c) \simeq \bar{\Gamma}_{\text{ID}}^c(z_{\text{dec}}^c) \Rightarrow \text{determines } \tilde{m}_{\text{od}}^{\text{min}}, \quad (3.10)$$

For $\tilde{m}_{\text{od}} \ll \tilde{m}_{\text{od}}^{\text{min}}$ LFE effects can be neglected, since they will attain thermal equilibrium only after leptogenesis is completed.

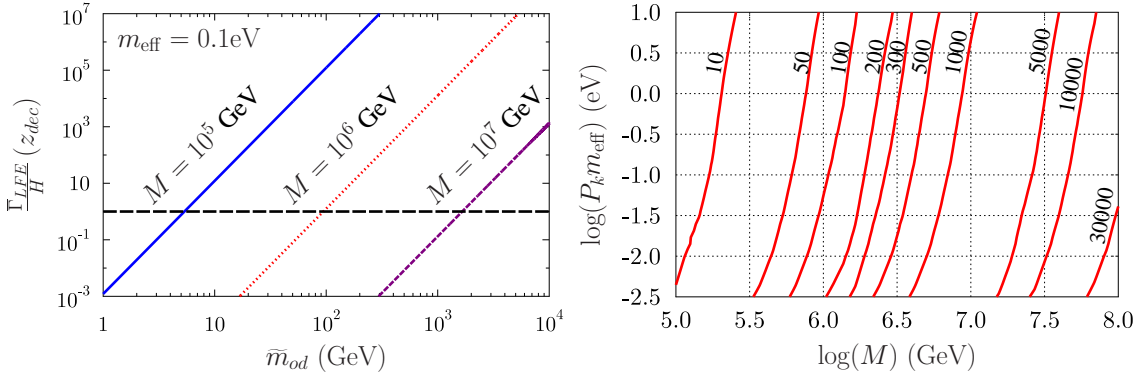


Figure I: The left panel shows the ratio of $\bar{\Gamma}_{\text{LFE}}$ to the Hubble expansion rate H at z_{dec} as a function of \tilde{m}_{od} for $m_{\text{eff}} = 0.1$ eV and $\tan \beta = 30$ and three values of M . The right panel shows in the $(P_k m_{\text{eff}}, M)$ plane, contours of constant values of \tilde{m}_{od} (in GeV) for which $\bar{\Gamma}_{\text{LFE}}(z_{\text{dec}}^k) \geq P_k \bar{\Gamma}_{\text{ID}}(z_{\text{dec}}^k)$.

In the right panel in Fig. I, we plot in the plane of the flavoured effective decay mass $P_k m_{\text{eff}}$ and of the RH sneutrino mass M , various contours corresponding to different values of \tilde{m}_{od} for which $\bar{\Gamma}_{\text{LFE}}(z_{\text{dec}}^k) = P_k \bar{\Gamma}_{\text{ID}}(z_{\text{dec}}^k)$. For a given value of M and m_{eff} , and for a given set of flavour projections $P_a < P_b < P_c$, $\tilde{m}_{\text{od}}^{\text{min}}$ is given by the value of the \tilde{m}_{od} curve for which the vertical line $x = M$ intersects the corresponding contour at $y_c = P_c m_{\text{eff}}$.

Furthermore, since $\bar{\Gamma}_{\text{LFE}}$ has a rather strong dependence on \tilde{m}_{od} ($\bar{\Gamma}_{\text{LFE}} \propto \tilde{m}_{\text{od}}^4$), one expects that the value $\tilde{m}_{\text{od}}^{\text{max}}$ for which LFE effects completely equilibrate the asymmetries in the different lepton flavours will not be much larger than $\tilde{m}_{\text{od}}^{\text{min}}$. Indeed our numerical results (see next section) show that $\tilde{m}_{\text{od}}^{\text{max}} \sim \mathcal{O}(5-10) \tilde{m}_{\text{od}}^{\text{min}}$. Clearly, as far as leptogenesis is concerned, larger values $\tilde{m}_{\text{od}} \gg \tilde{m}_{\text{od}}^{\text{max}} \sim \tilde{m}_{\text{od}}^{\text{min}}$ do not imply any modification in the numerical results with respect to what is obtained with $\tilde{m}_{\text{od}} = \tilde{m}_{\text{od}}^{\text{max}}$.

4. Results

We quantify the results that can be obtained by including LFE effects by solving the

following set of BE for the abundances $Y_X = n_X/s$:

$$-sHz \frac{dY_N}{dz} = \left(\frac{Y_N}{Y_N^{eq}} - 1 \right) \left(\gamma_N + 4\gamma_t^{(0)} + 4\gamma_t^{(1)} + 4\gamma_t^{(2)} + 2\gamma_t^{(3)} + 4\gamma_t^{(4)} \right), \quad (4.1)$$

$$-sHz \frac{dY_{\tilde{N}_{tot}}}{dz} = \left(\frac{\gamma_{\tilde{N}}}{2} + \gamma_{\tilde{N}}^{(3)} + 3\gamma_{22} + 2\gamma_t^{(5)} + 2\gamma_t^{(6)} + 2\gamma_t^{(7)} + \gamma_t^{(8)} + 2\gamma_t^{(9)} \right) \left(\frac{Y_{\tilde{N}_{tot}}}{Y_{\tilde{N}}^{eq}} - 2 \right) \quad (4.2)$$

$$\begin{aligned} -sHz \frac{dY_{\Delta_{tot}^k}}{dz} = & \epsilon^k(T) \frac{\gamma_{\tilde{N}}}{2} \left(\frac{Y_{\tilde{N}_{tot}}}{Y_{\tilde{N}}^{eq}} - 2 \right) - \left[\frac{\gamma_{\tilde{N}}^k}{2} + \frac{\gamma_N^k}{2} + \gamma_{\tilde{N}}^{(3)k} + \left(\frac{1}{2} \frac{Y_{\tilde{N}_{tot}}}{Y_{\tilde{N}}^{eq}} + 2 \right) \gamma_{22}^k \right] \left(\frac{Y_{L_{tot}^k}}{Y_{L_{tot}}^c} + \frac{Y_{H_{tot}}}{Y_{H_{tot}}^c} \right) \\ & - 2 \left(\gamma_t^{(1)k} + \gamma_t^{(2)k} + \gamma_t^{(4)k} + \gamma_t^{(6)k} + \gamma_t^{(7)k} + \gamma_t^{(9)k} \right) \frac{Y_{L_{tot}^k}}{Y_{L_{tot}}^c} \\ & - \left[\left(2\gamma_t^{(0)} + \gamma_t^{(3)k} \right) \frac{Y_N}{Y_N^{eq}} + \left(\gamma_t^{(5)k} + \frac{1}{2} \gamma_t^{(8)k} \right) \frac{Y_{\tilde{N}_{tot}}}{Y_{\tilde{N}}^{eq}} \right] \frac{Y_{L_{tot}^k}}{Y_{L_{tot}}^c} \\ & - \left(2\gamma_t^{(0)k} + \gamma_t^{(1)k} + \gamma_t^{(3)k} + \gamma_t^{(4)k} + 2\gamma_t^{(5)k} + \gamma_t^{(6)k} + \gamma_t^{(7)k} + \gamma_t^{(8)k} + \gamma_t^{(9)k} \right) \frac{Y_{H_{tot}}}{Y_{H_{tot}}^c} \\ & - \left[\left(\gamma_t^{(1)k} + \gamma_t^{(2)k} + \gamma_t^{(4)k} \right) \frac{Y_N}{Y_N^{eq}} + \frac{1}{2} \left(\gamma_t^{(6)k} + \gamma_t^{(7)k} + \gamma_t^{(9)k} \right) \frac{Y_{\tilde{N}_{tot}}}{Y_{\tilde{N}}^{eq}} \right] \frac{Y_{H_{tot}}}{Y_{H_{tot}}^c} \\ & - 84 \sum_{j \neq k} \left(\gamma_{t1,2}^{jk} + \gamma_{t2,2}^{jk} + \gamma_{s,2}^{jk} \right) \frac{Y_{L_{tot}^k} - Y_{L_{tot}^j}}{Y_{L_{tot}}^c} \\ & - 72 \sum_{j \neq k} \left(\gamma_{t1,Y}^{jk} + \gamma_{t2,Y}^{jk} + \gamma_{s,Y}^{jk} \right) \frac{Y_{L_{tot}^k} - Y_{L_{tot}^j}}{Y_{L_{tot}}^c}. \end{aligned} \quad (4.3)$$

Spectator effects and sphaleron flavour mixing are taken into account by writing

$$Y_{L_{tot}^k} = \sum_j A_{kj} Y_{\Delta_{tot}^j}, \quad Y_{H_{tot}} = \sum_j C_j Y_{\Delta_{tot}^j}. \quad (4.4)$$

In Eqs. (4.1)-(4.3) we have defined $Y_{\Delta_{tot}^k} \equiv Y_B/3 - Y_{L_{tot}^k}$, $Y_{\tilde{N}_{tot}} \equiv Y_{\tilde{N}_+} + Y_{\tilde{N}_-}$, and $Y_{L_{tot}^k} \equiv Y_{L_f^k} + Y_{L_s^k}$ that is the total asymmetry in flavour k obtained by summing up both fermions $Y_{L_f^k}$ and scalars $Y_{L_s^k}$ contributions, where for example $Y_{L_f^k} = (Y_{\ell_k} - Y_{\bar{\ell}_k})$, while $Y_{H_{tot}}$ is the total asymmetry for the Higgs and Higgsinos. In addition we have $Y_{H_{tot}}^c = Y_{L_{tot}}^c \equiv \frac{45}{4\pi^2 g^*}$ and $Y_{\tilde{N}}^{eq}(T \gg M) = 90\zeta(3)/(4\pi^4 g^*)$.

The values of the entries of the matrix A and of the vector C in Eq. (4.4) depend on the range of temperature, that is on the particular set of interactions that are in equilibrium when leptogenesis is taking place. For $T < (1 + \tan^2 \beta) \times 10^5 \text{ GeV}$ reactions mediated by the Yukawa couplings of all the three families are in equilibrium [2], and in this case the A

and C matrices are given by [§]

$$A = \frac{2}{711} \begin{pmatrix} -221 & 16 & 16 \\ 16 & -221 & 16 \\ 16 & 16 & -221 \end{pmatrix}, \quad C = -\frac{8}{79} (1 \ 1 \ 1). \quad (4.5)$$

Note that the last two lines in Eq.(4.3) correspond to the reaction densities for the LFE processes given in Eq.(3.6), and play the role of controlling the effectiveness of the leptogenesis flavour effects. The different reaction densities for the $\Delta L = 1$ processes are:

$$\begin{aligned} \gamma_{\tilde{N}}^k &= \sum_{i=\pm} \left[\gamma(\tilde{N}_i \leftrightarrow \bar{h}\ell_k) + \gamma(\tilde{N}_i \leftrightarrow h\bar{\ell}_k) \right], \\ \gamma_{\tilde{N}}^{(3)k} &= \sum_{i=\pm} \gamma(\tilde{N}_i \leftrightarrow \tilde{\ell}_k^* \tilde{u}\tilde{q}), \\ \gamma_{22}^k &= \sum_{i=\pm} \gamma(\tilde{N}_i \tilde{\ell}_k \leftrightarrow \tilde{u}\tilde{q}) = \sum_{i=\pm} \gamma(\tilde{N}_i \tilde{q}^* \leftrightarrow \tilde{\ell}_k^* \tilde{u}) = \sum_{i=\pm} \gamma(\tilde{N}_i \tilde{u}^* \leftrightarrow \tilde{\ell}_k^* \tilde{q}), \\ \gamma_t^{(5)k} &= \sum_{i=\pm} \gamma(\tilde{N}_i \ell_k \leftrightarrow q\tilde{u}) = \sum_{i=\pm} \gamma(\tilde{N}_i \ell_k \leftrightarrow \bar{q}\tilde{u}), \\ \gamma_t^{(6)k} &= \sum_{i=\pm} \gamma(\tilde{N}_i \tilde{u}^* \leftrightarrow \bar{\ell}_k q) = \sum_{i=\pm} \gamma(\tilde{N}_i \tilde{q}^* \leftrightarrow \bar{\ell}_k \tilde{u}), \\ \gamma_t^{(7)k} &= \sum_{i=\pm} \gamma(\tilde{N}_i \bar{q} \leftrightarrow \bar{\ell}_k \tilde{u}) = \sum_{i=\pm} \gamma(\tilde{N}_i u \leftrightarrow \bar{\ell}_k \tilde{q}), \\ \gamma_t^{(8)k} &= \sum_{i=\pm} \gamma(\tilde{N}_i \tilde{\ell}_k^* \leftrightarrow \bar{q}u), \\ \gamma_t^{(9)k} &= \sum_{i=\pm} \gamma(\tilde{N}_i q \leftrightarrow \tilde{\ell}_k u) = \sum_{i=\pm} \gamma(\tilde{N}_i \bar{u} \leftrightarrow \tilde{\ell}_k \bar{q}), \\ \gamma_N^k &= \gamma(N \leftrightarrow \ell_k h) + \gamma(N \leftrightarrow \tilde{\ell}_k^* \tilde{h}), \\ \gamma_t^{(0)k} &= \gamma(N \tilde{\ell}_k \leftrightarrow q\tilde{u}) = \gamma(N \tilde{\ell}_k \leftrightarrow \bar{q}\tilde{u}), \\ \gamma_t^{(1)k} &= \gamma(N \bar{q} \leftrightarrow \tilde{\ell}_k^* \tilde{u}) = \gamma(N u \leftrightarrow \tilde{\ell}_k^* \tilde{q}), \\ \gamma_t^{(2)k} &= \gamma(N \tilde{u}^* \leftrightarrow \tilde{\ell}_k^* q) = \gamma(N \tilde{q}^* \leftrightarrow \tilde{\ell}_k^* \bar{u}), \\ \gamma_t^{(3)k} &= \gamma(N \ell_k \leftrightarrow q\bar{u}), \\ \gamma_t^{(4)k} &= \gamma(N u \leftrightarrow \bar{\ell}_k q) = \gamma(N \bar{q} \leftrightarrow \bar{\ell}_k \bar{u}). \end{aligned} \quad (4.6)$$

When no flavour index appears in the γ 's, as in Eqs. (4.1) and (4.2), it is understood that the corresponding reactions have been summed over all lepton flavours. The flavoured and flavour-summed rates are thus related according to

$$\gamma_X^k = P_k \gamma_X. \quad (4.7)$$

In our calculation we have kept particle thermal masses and fermion-boson statistical factors only in the CP asymmetries, but we have neglected them in the rest of the reaction

[§]Indeed we find that, within a given T regime, A and C for the MSSM and for the SM are the same up to a global factor 1/2 for C . This is expected to be so, since supersymmetry cannot alter the flavour distribution between the charges. This is in agreement with the analysis in Ref. [32], but it disagrees with the A matrix given in Ref. [33].

rates, with the exception of the thermal Higgs mass in the $\Delta L = 1$ processes involving a Higgs boson exchange in the t -channel, that is needed to regulate the IR divergence that occurs in the limit $m_h \rightarrow 0$.

In what follows we consider the resonant (self-energy) CP asymmetry $\epsilon_k^S(T)$. We parametrize the asymmetry generated by the decay of the singlet sneutrino states in a given flavour as

$$Y_{\Delta_{tot}^k}(z \rightarrow \infty) = -2\eta_k \bar{\epsilon} Y_{\tilde{N}}^{\text{eq}}(T \gg M) \quad (4.8)$$

where, to facilitate comparison with the existing literature, we have defined

$$\bar{\epsilon} = -\sin \phi_A \frac{A}{M} \frac{4B\Gamma}{4B^2 + \Gamma^2} . \quad (4.9)$$

The final amount of $B - L$ asymmetry generated in the decay of the singlet sneutrinos (we assume no pre-existing asymmetry) can be parametrized as:

$$Y_{B-L}(z \rightarrow \infty) = \sum_k Y_{\Delta_{tot}^k}(z \rightarrow \infty) = -2\eta \bar{\epsilon} Y_{\tilde{N}}^{\text{eq}}(T \gg M), \quad (4.10)$$

with

$$\eta = \sum_k \eta_k. \quad (4.11)$$

After conversion by the sphaleron transitions, the final baryon asymmetry is related to the $B - L$ asymmetry by

$$Y_B = \frac{24 + 4n_h}{66 + 13n_h} Y_{B-L}(z \rightarrow \infty) = \frac{8}{23} Y_{B-L}(z \rightarrow \infty), \quad (4.12)$$

where n_h is the number of Higgs doublets, and in the second equality we have taken $n_h = 2$.

According to our expressions for the CP asymmetries Eqs. (2.9)-(2.11) and to the general expression for the flavoured reaction rates Eq. (4.7), and neglecting for the time being LFE effects, η_k depends on flavour via the projections P_k and Z_k/Y_k . The final asymmetry produced also depends on the Yukawa couplings $\sum_j Y_j^2$ and on the heavy singlet mass M through the combination m_{eff} defined in Eq. (2.8) (there is a residual mild dependence on M due to the running of the top Yukawa coupling). The dependence of the efficiency factor on the flavour projections and on m_{eff} is shown in Fig.II. The results are obtained assuming that the \tilde{N} population is created by their Yukawa interactions with the thermal plasma, that is $Y_{\tilde{N}}(z \rightarrow 0) = 0$, and neglecting for the time being the possible effects of LFE. The plot is shown for $M = 10^6$ GeV and $\tan \beta = 30$ although, as mentioned above, the efficiency is practically independent of M . As long as $\tan \beta$ is not very close to one, the dominant dependence on $\tan \beta$ arises via v_u as given in Eq. (2.8) and it is therefore also rather mild. For $\tan \beta \sim \mathcal{O}(1)$ there is also an additional (very weak) dependence due to the associated change in the top Yukawa coupling. The results are displayed for the two choices of soft-breaking terms that have been discussed at the end of Sec. 2: the UTS, defined by Eq.(2.23), and our SMS, defined by Eq.(2.25). We note that these two scenarios are equivalent for the special case of flavour equipartition $P_1 = P_2 = P_3 = 1/3$.

From the left panel in Fig.II we see that departure from the equipartition flavour case results in an enhancement of the efficiency, and that particularly large enhancements

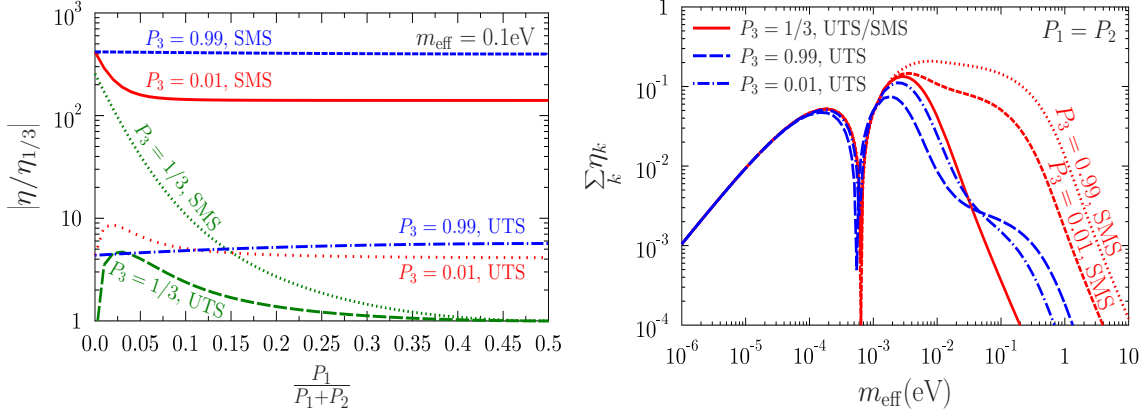


Figure II: The dependence of the efficiency – normalized to the flavour equipartition case $P = (1/3, 1/3, 1/3)$ – on the values of the lepton flavour projections (left) and on m_{eff} (right). The figures correspond to $M = 10^6$ GeV and $\tan \beta = 30$.

are possible for the SMS scenario. Note that the top line in the left panel of Fig. II labeled $P_3 = 0.99$ represents the maximum enhancement that can be obtained in the SMS (relaxing the constraint in Eq. (2.25) that defines our SMS, larger enhancements are however possible). This is because for $P_3 = 0.99$ both the asymmetries $Y_{\Delta_{\text{tot}}}^1$ and $Y_{\Delta_{\text{tot}}}^2$ are generated in the weak washout regime, that is, approximately within the same temperature range, and in the SMS this implies $\epsilon_1(T_1) \approx \epsilon_2(T_2)$. The related combined efficiency is then simply determined by $(P_1 + P_2) m_{\text{eff}} \simeq m_*$ and is thus always maximal, independently of the individual values of P_1 and P_2 , as is apparent from the figure.

The right panel of Fig. II shows the dependence of the efficiency on m_{eff} in the flavour equipartition case and for two other sets of flavour projections. As it is known, flavour effects become more relevant when the washouts get stronger. This is confirmed in this picture where it is seen that for the SMS scenario the possible enhancements quickly grow with m_{eff} . Note that in soft leptogenesis this dependence is even stronger than in standard leptogenesis. This is due to the fact that the flavoured washout parameters $P_k m_{\text{eff}}$ also determine the value of z_{dec}^k when the lepton asymmetry in the k flavour starts being generated, and since the CP asymmetry has a strong dependence on z , different values of P_1 , P_2 , and P_3 imply that the corresponding flavour asymmetries are generated with different values of the CP asymmetry even when, as in the SMS, the fundamental quantity $\bar{\epsilon}$ is flavour independent. In summary, what happens is that the flavour that suffers the weakest washout is also the one for which inverse-decays go out of equilibrium earlier, and thus also the one for which the lepton asymmetry starts being generated when $\bar{\epsilon} \times \Delta_{BF}$ is larger. This realizes a very efficient scheme in which the flavour that is more weakly washed out has effectively the largest CP asymmetry, and this explains qualitatively the origin of the large enhancements that we have found. Furthermore, when $P_k m_{\text{eff}} \ll m_*$ so that the inverse decay of flavour k never reaches equilibrium and the washout of the asymmetry $Y_{\Delta_{\text{tot}}}^k$ is negligible, the maximum efficiency is reached.

We should however spend a word of caution for the reader about interpreting our

numerical results in the weak washout regime and, for the SMS, also in the limit of extreme flavour hierarchies ($P_k \rightarrow 0$). At high temperatures ($z < 1$) the Higgs bosons (higgsinos) develop a sufficiently large thermal mass to decay into sleptons (leptons) and sneutrinos. The new CP asymmetries associated with these decays could be particularly large [30], and thus sizable lepton flavour asymmetries could be generated at high temperatures. This type of thermal effects are not included in our analysis. Concerning the flavour decoupling limit within the SMS, clearly when $P_k \rightarrow 0$ no asymmetry can be generated in the flavour k . However, in our SMS flavour asymmetries are defined to be independent of the projectors P and thus survive in the $P \rightarrow 0$ limit. On physical grounds, one would expect for example that when one decay branching ratio is suppressed, say, as $P < 10^{-5}$, the associated CP asymmetry will be at most of $\mathcal{O}(10^{-7})$ and thus irrelevant for leptogenesis. This means that for extreme flavour hierarchies, the SMS breaks down as a possible physical realization of soft leptogenesis, and thus in what follows we will restrict our considerations to a range of hierarchies $P \gtrsim 10^{-3}$.

As a result of our analysis, we find that for the SMS scenario with hierarchical Yukawa couplings, successful leptogenesis is possible even for $m_{\text{eff}} \gg \mathcal{O}(\text{eV})$. For example, as is shown in the right panel of Fig. II, for $P_1 = P_2 = 5 \times 10^{-3}$ and $m_{\text{eff}} \sim 5 \text{ eV}$, we obtain $|\eta| \sim 10^{-3}$, that yields the estimate

$$Y_B(\text{SMS}, P_1 = P_2 = 5 \times 10^{-3}, m_{\text{eff}} = 5 \text{ eV}) \sim 10^{-6} \times \bar{\epsilon}. \quad (4.13)$$

Thus we see that assuming a large, but still acceptable value of $\bar{\epsilon} \sim 10^{-4}$, soft leptogenesis can successfully generate the observed baryon asymmetry [34]:

$$Y_{B_{\text{obs}}} = (8.78 \pm 0.24) \times 10^{-11} \quad (4.14)$$

also for values of m_{eff} that are about two orders of magnitude larger than what is found in the unflavoured standard leptogenesis scenario.

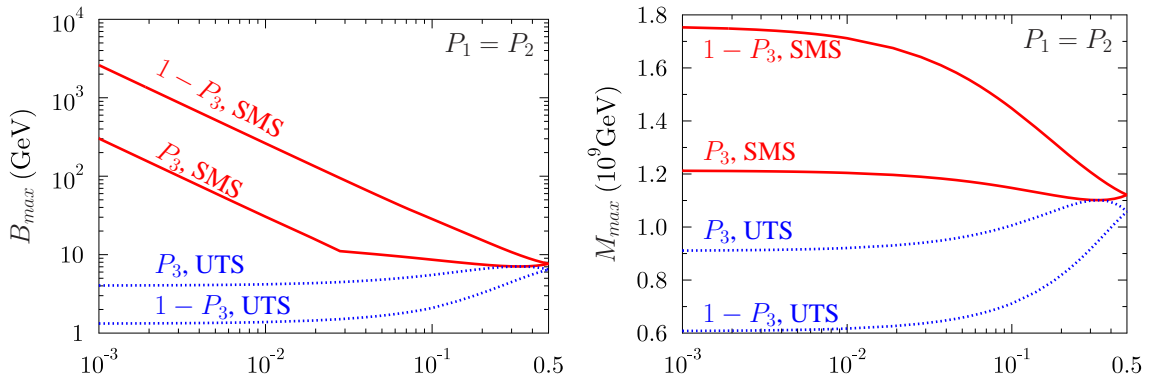


Figure III: Maximum values of B and M which can lead to successful leptogenesis as a function of the flavour projections (we plot both as a function of P_3 or $1 - P_3$ for clarity when either P_3 or $1 - P_3$ is very small). The figure corresponds to $A \sin \phi_A = 1 \text{ TeV}$ and $\tan \beta = 30$.

We next explore the impact that flavour enhancements can have in relaxing the requirements on the values of B and M for successful leptogenesis. From Eqs. (4.10), (4.12), (4.9),

and (4.14) we find that the maximum value of B for given values of M and m_{eff} is:

$$B \leq \frac{\Gamma(m_{\text{eff}}, M)}{2} \frac{|\text{Im}A|}{M} \frac{C \eta(m_{\text{eff}})}{Y_{B_{\text{obs}}}} \left[1 + \sqrt{1 - \left(\frac{M}{|\text{Im}A|} \frac{Y_{B_{\text{obs}}}}{C \eta(m_{\text{eff}})} \right)^2} \right], \quad (4.15)$$

where $C = \frac{16}{23} Y_{\tilde{N}}^{eq}(z \rightarrow 0)$, $\Gamma(m_{\text{eff}}, M)$ is given in Eq.(2.8) and $\text{Im}A = A \sin \phi_A$. Thus we obtain

$$M \leq \frac{|\text{Im}A| C \eta(m_{\text{eff}})}{Y_{B_{\text{obs}}}}, \quad (4.16)$$

$$B \leq \frac{3\sqrt{3}m_{\text{eff}}}{32\pi v^2} \left(\frac{|\text{Im}A| C \eta(m_{\text{eff}})}{Y_{B_{\text{obs}}}} \right)^2, \quad (4.17)$$

where $\eta(m_{\text{eff}}) \equiv \eta(m_{\text{eff}}, P_j, Z_j)$ and we have neglected all residual dependence of η on M . As seen in the right panel of Fig. II, assuming the SMS and for sufficiently hierarchical P_j , $\eta(m_{\text{eff}})$ decreases first very mildly with m_{eff} and – once all the flavours have reached the strong washout regime – it decreases roughly as $\sim m_{\text{eff}}^{-2}$. Thus the product $m_{\text{eff}} \times \eta(m_{\text{eff}})^2$ first grows with m_{eff} till it reaches a maximum and then for sufficiently large m_{eff} it decreases $\sim m_{\text{eff}}^{-3}$. Therefore, for a fixed value of the projectors, the upper bound on B does not corresponds simply to the maximum allowed value of m_{eff} , but it has a more complicated dependence.

In Fig. III we show the maximum values of B and M obtained for both the UTS and SMS cases as a function of the flavour projections. In order to have better resolution when either P_3 or $1 - P_3$ is very small, we plot them both as a function of P_3 or $1 - P_3$. In the figure we set $\text{Im}A = 1$ TeV. The figure illustrates that within the UTS, the parameter space for successful leptogenesis is very little modified by departing from the flavour equipartition case (that corresponds to the point where the UTS and SMS curves join). On the contrary, in the SMS case we find that with hierarchical flavour projections $1 - P_3 \sim \text{few} \times 10^{-3}$ successful soft-leptogenesis is allowed also with $B \sim \mathcal{O}(\text{TeV})$, that is for quite natural values of the bilinear term. As mentioned above, even for hierarchical projections the maximum allowed values of B and M do not correspond to the maximum allowed value of m_{eff} . In particular, for the range of flavour projections shown in the figure we obtain that the maximum values of B and M correspond to $m_{\text{eff}} \lesssim 2$ eV.

We now turn to quantify the impact that the presence of LFE scatterings can have on these results. We plot in Fig.IV the dependence of the enhancement of the efficiency due to flavour effects, as a function of the off-diagonal slepton mass parameter \tilde{m}_{od} . As can be seen in the figure (and as it was expected from the discussion in the previous section) for any given value of M , LFE quickly becomes efficient damping completely the lepton flavours enhancements of the efficiency within a very narrow range of values $\tilde{m}_{od}^{\text{min}} \leq \tilde{m}_{od} \leq \tilde{m}_{od}^{\text{max}}$. The figure is shown for $\tan \beta = 30$. Again, the dominant dependence on $\tan \beta$ arises due to $v_d = v \cos \beta$ in Eq.(3.5). Results from other values of $\tan \beta$ can be easily read from the figure by rescaling $\tilde{m}_{od}^\beta = \tilde{m}_{od}^{\text{fig}} / (30 \cos \beta)$.

It is interesting to remark that while the efficiency $\eta(m_{\text{eff}})$ is practically insensitive to the particular value of M , as long as m_{eff} is held constant, this is not the case for the

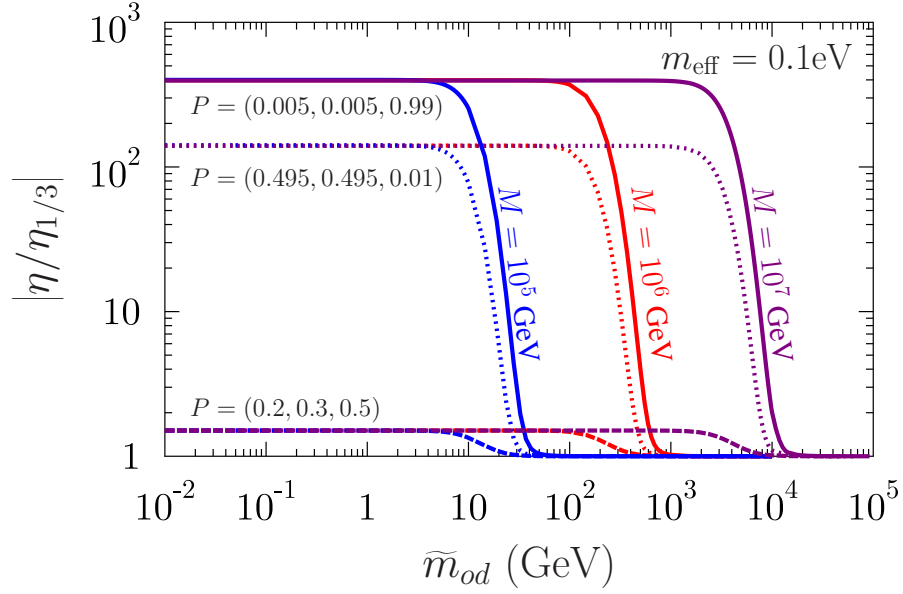


Figure IV: The dependence of the efficiency (normalized to the flavour equipartition case $P = (1/3, 1/3, 1/3)$) on the off-diagonal soft slepton mass parameter \tilde{m}_{od} , for different values of M and of the flavour projections (see text for details).

generalized efficiency η^{LFE} computed by accounting for LFE effects. Given the different scaling with the temperature of the $\bar{\Gamma}_{\text{LFE}}$ and $\bar{\Gamma}_{\text{ID}}$ rates, the precise temperature at which leptogenesis occurs is crucial. For example, we see from Fig.IV that for reasonable values $\tilde{m}_{od} \lesssim 200 \text{ GeV}$ and for $M \gtrsim 10^6 \text{ GeV}$, LFE is not effective, and the large enhancements of the efficiency due to flavour effects can survive, while for $M \lesssim 10^5 \text{ GeV}$ all flavour enhancements disappear.

5. Low energy constraints

We have seen in the previous section that for not too large values $\tilde{m}_{od} \sim 200 \text{ GeV}$ and for $M \gtrsim 10^6 \text{ GeV}$ LFE effects can be neglected. In this section we address this issue more quantitatively, comparing the values of \tilde{m}_{od} required for total washout of flavour effects in soft leptogenesis, with the bounds imposed from non-observation of flavour violation in leptonic decays. The question we want to address is the following: given the low energy constraints on \tilde{m}_{od} , what is the lower bound on the leptogenesis scale M for which large flavour enhancements of the lepton asymmetry are not damped by LFE effects ?

Clearly the presence of a sizable \tilde{m}_{od} would induce various LFV decays, like for example $l_j \rightarrow l_k \gamma$ with rate

$$\frac{BR(l_j \rightarrow l_k \gamma)}{BR(l_j \rightarrow l_k \nu_j \bar{\nu}_k)} \sim \frac{\alpha^3}{G_F^2} \frac{\tan^2 \beta}{m_{SUSY}^8} \tilde{m}_{od}^4 \simeq 2.9 \times 10^{-19} \frac{\sin^2 \beta}{\cos^6 \beta} \left(\frac{\text{TeV}}{m_{SUSY}} \right)^8 \left(\cos^2 \beta \frac{\tilde{m}_{od}^2}{\text{GeV}^2} \right)^2 \quad (5.1)$$

where m_{SUSY} is a generic SUSY scale for the gauginos and sleptons masses running in the LFV loop. We show in Fig.V with a yellow shade, the excluded region of $\tilde{m}_{od} \cos \beta$

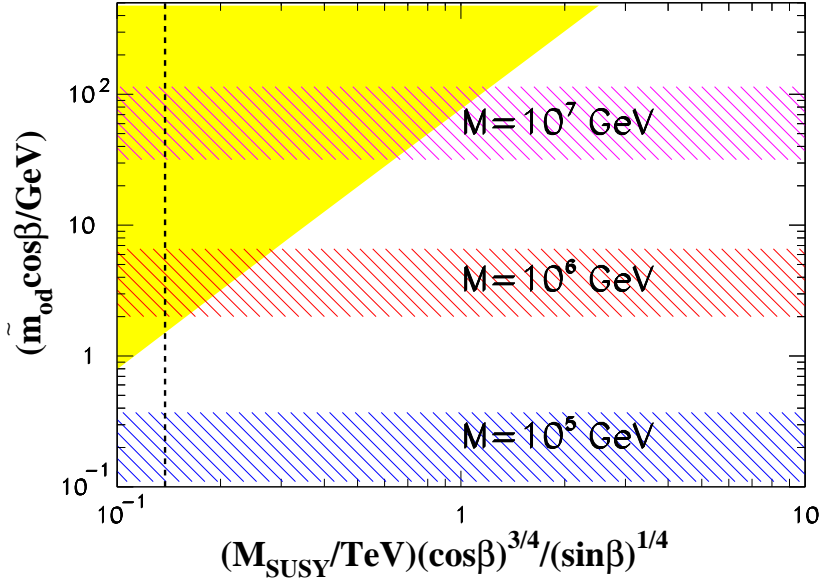


Figure V: Excluded region (shaded in yellow) of $\tilde{m}_{od} \cos \beta$ versus $m_{\text{SUSY}}(\cos \beta)^{3/4}/(\sin \beta)^{1/4}$ arising from the present bound of $BR(\mu \rightarrow e\gamma) \leq 1.2 \times 10^{-11}$, together with the minimum value of $\tilde{m}_{od} \cos \beta$ for which LFE effects start damping out flavour effects in soft-leptogenesis. Three bands are shown corresponding to $M = 10^5$ GeV, $M = 10^6$ GeV and $M = 10^7$ GeV. The width of the bands represents the range associated with variations of $P_k m_{eff}$ in the range 0.003 eV–10 eV, where P_k is the largest of the three flavour projections. The vertical dashed line represents the value of $m_{\text{SUSY}}/(\tan \beta)^{1/2}$ (for $\tan \beta = 1$) required to explain the discrepancy between the SM prediction and the measured value of a_μ [35].

versus $m_{\text{SUSY}}(\cos \beta)^{3/4}/(\sin \beta)^{1/4}$ arising from the present bound $BR(\mu \rightarrow e\gamma) \leq 1.2 \times 10^{-11}$, together with the minimum value of $\tilde{m}_{od} \cos \beta$ for which LFE effects start damping out flavour enhancements in soft-leptogenesis. Three bands are shown respectively for $M = 10^5$ GeV, $M = 10^6$ GeV and $M = 10^7$ GeV. The width of the bands represents the range associated with variations of the effective flavoured decay parameter $P_k m_{eff}$ in the range 0.003 eV–10 eV, where P_k is the largest of the three flavour projections. For illustration we also show in the figure the characteristic SUSY scale that allows to explain the small discrepancy between the SM prediction and the measured value of the muon anomalous magnetic moment, a_μ . This value is $m_{\text{SUSY}}/(\tan \beta)^{1/2} = 141$ GeV [35], and the vertical dashed line in the picture corresponds to $\tan \beta = 1$. As seen in the figure, in this case the off-diagonal slepton masses are bound to be small enough to allow for flavour enhancements in soft leptogenesis for M as low as 10^6 GeV. For larger values of $\tan \beta$, even lower values of M are allowed.

6. Discussion and Conclusions

Within the supersymmetric leptogenesis scenario, the generation of the baryon asymmetry unavoidably receives contributions from dynamical effects in the decays of the heavy sneutrinos that are specifically related to soft SUSY breaking terms. The interesting point is that soft leptogenesis effects become relevant at relatively low temperatures, and precisely when the usual contributions surviving in the limit of unbroken supersymmetry become ineffective to generate a sufficient amount of lepton asymmetry. Soft leptogenesis thus opens up a low temperature window where supersymmetric leptogenesis can proceed without conflicting with the gravitino limits on the reheating temperature.

However, the efficiency of soft leptogenesis remains relatively low, especially when flavour effects are neglected. This is mainly due to the fact that the relevant CP asymmetries vanish in the zero temperature limit. Some mechanism to generate sufficiently large enhancements of the lepton asymmetry that is produced are then required, and are generally obtained by assuming a resonant or quasi-resonant regime for the decays of the pair of heavy sneutrinos belonging to the same family. Rather unpleasantly, the resonant conditions can be ensured only by requiring that the sneutrino mixing parameter B , that is the parameter that controls the splitting between the pairs of mass eigenvalues, has a value that is unnaturally suppressed with respect to the SUSY breaking scale: $B \ll m_{SUSY}$.

However, given the temperature regimes in which soft leptogenesis can proceed, accounting for flavour effects is mandatory. These effects were first studied in ref. [16] under the assumption of universality of the soft SUSY breaking terms. Such scenarios strongly constrain the possible flavour structures, and in particular imply that the flavoured CP asymmetries must be proportional to the corresponding flavour dependent washouts, with the result that the larger is the CP asymmetry, the more efficient is the related washout. This compensating mechanism allows for only moderate $\sim \mathcal{O}(30)$ enhancements of the leptogenesis efficiency. Thus, within universal soft-breaking schemes, flavour effects can only moderately alleviate the fine tuning problem of the B parameter, and still do not allow for $B \sim m_{SUSY}$, that is what one would expect on the basis of naturalness considerations.

In this paper we have shown how this situation drastically changes if the assumption of universality for the soft-breaking terms is relaxed, which results in a generic situation in which the flavoured CP asymmetries are not aligned with the respective washouts. Note that an analogous situation is generally realized within the standard flavoured leptogenesis scenarios. To carry out our phenomenological analysis, while avoiding the proliferation of too many flavour-related parameters, we have introduced a simplified non-universal scheme in which all the flavoured CP asymmetries (evaluated at equal temperatures) are equal (and thus flavour independent) while the flavoured washouts are allowed to be strongly hierarchical. Here we stress that since the hierarchy in the washouts is controlled by the hierarchy in the sneutrino Yukawa couplings, and given that we know that in the SM strong hierarchies in the Yukawa couplings are realized in the charged lepton sector as well as for the up- and down-type quark sectors, a strong hierarchy in the flavour dependent washouts can be considered as a natural possibility. As regards the amount of misalignment between the soft-breaking trilinear terms and the corresponding Yukawa couplings, that eventually

produces the misalignment between flavoured CP asymmetries and washouts, due to our ignorance about the mechanism that breaks SUSY, any assumption is equally acceptable, provided that the existing limits on LFV processes are not violated.

Under these conditions, we have found that flavour effects can enhance the leptogenesis efficiency by more than two orders of magnitude with respect to the flavour equipartition case, defined as the situation in which all the flavoured CP asymmetries and washouts are equal in magnitude. This result can then be translated into a several $\times 10^3$ enhancement with respect to the one-flavour approximation, which is sufficient to avoid the need for any additional enhancement from resonant conditions. Thus, the natural scale for the sneutrino mixing parameter $B \sim m_{SUSY}$ is eventually allowed. Curiously, the possibility of such large enhancements is directly related to the strong temperature dependence of the CP asymmetries: for the lepton flavours that are most weakly washed out, and for which inverse-decays go out of equilibrium first, the lepton asymmetry is generated at larger temperatures, that is precisely where the CP asymmetry is larger. Thus, relying only on the assumption of flavour misalignment and of hierarchical Yukawa couplings, a very efficient scheme in which the weaker is the washout, the larger is the corresponding CP asymmetry, is automatically realized, and this boosts the leptogenesis efficiency to the highest possible values.

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